

Math 2FM3, Tutorial 6

Oct 26th, 2015

Outstanding Balance

- Set the initial outstanding balance

$$OB_0 = K_1 v + K_2 v^2 + \dots + K_n v^n$$

- After first payment K_1

$$OB_1 = OB_0 (1+i) - K_1$$

- After second payment K_2

$$OB_2 = OB_1 (1+i) - K_2 = OB_0 (1+i)^2 - K_1(1+i) - K_2$$

- After t^{th} payment K_t

$$OB_t = OB_0 (1+i)^t - K_1(1+i)^{t-1} - \dots - K_{t-1} (1+i) - K_t$$

Retrospective and Prospective form

- If all of the payments are identical, $K_1 = K_2 = \dots = K_t = K$,

$OB_t = OB_0 (1+i)^t - Ks_{t|i}$ is retrospective form

$OB_t = Ka_{n-t|i}$ is prospective form

Interest paid and Principal repaid

- Interest paid:

$$I_t = OB_{t-1} \cdot i$$

- Principal repaid:

$$\begin{aligned} PR_t &= OB_{t-1} - OB_t = OB_{t-1} - OB_{t-1}(1+i) + K_t \\ &= K_t - OB_{t-1} \cdot i = K_t - I_t \end{aligned}$$

Ex 3.1.1

- An amortized loan has 10 annual payments at the end of each year starting one year from now. The first 5 payments are \$1000 each and the final 5 payments are \$500 each. Interest is at an effective annual rate of 10%. Find each of the following:
 - (i) the initial loan amount
 - (ii) the outstanding balance just after the 3rd payment
 - (iii) the interest and principal in the 4th payment
 - (iv) the outstanding balance just after the 8th payment.

Ex 3.1.4

- Smith borrows 20,000 to purchase a car. The car dealer finances the purchase and offers Smith two alternative financing plans, both of which require monthly payments at the end of each month for 4 years starting one month after the car is purchased.
- (i) 0% interest rate for the first year followed by 6% nominal annual interest rate compounded monthly for the following three years.
- (ii) 3% nominal annual interest rate compounded monthly for the first year followed by 5% nominal annual interest compounded monthly for the following three years.
- For each of (i) and (ii) find the monthly payment and the outstanding balance on the loan at the end of the first year.

Ex 3.2.5

- (a) A 5-year loan is amortized with semiannual payments of 200 each, starting 6 months after the loan is made. If $PR_1 = 156.24$, find $i^{(12)}$.
- (b) A loan is repaid by 48 monthly payments of 200 each. The interest paid in the first 12 payments is 983.16 and the principal repaid in the final 12 payments is 2215.86. Find $i^{(12)}$.